DOE/ER/40561–251–INT96–00–120

March 1996

Small x parton distributions and initial conditions in ultrarelativistic nuclear collisions.

Raju Venugopalan

Institute for Nuclear Theory, University of Washington, Seattle, WA 98195

Abstract

At the colliders RHIC and LHC, nuclei at the ultrarelativistic energies of 100 GeV/A and 2.7 TeV/A will be smashed together with the hope of creating an elusive and short-lived state of matter called the quark gluon plasma. The initial conditions which determine the dynamical evolution of the quark gluon matter formed in the central region after the collision depend crucially on the small x component of the nuclear wavefunction before the collision. In this comment, we discuss recent work which argues that, for large nuclei, weak coupling techniques in QCD can be used to calculate the distribution of these small x, or wee, partons. The ramifications of this approach for the dynamics of heavy ion collisions and the various signatures of a quark gluon phase of matter are discussed.

1 Introduction

What does a nucleus look like when it is boosted to ultrarelativistic energies? The special theory of relativity tells us that the nucleus must contract a distance R/γ in direction of its motion, where R is its radius and $\gamma >> 1$ is the Lorentz factor. If we increase γ indefinitely, do we expect the longitudinal size of the nucleus to shrink to a point? What does this statement mean in terms of the underlying parton degrees of freedom? What happpens to its transverse size–does it approach a constant at asymptotic energies or does it keep growing [1]?

With the advent of the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC), which will collide large nuclei at ultrarelativistic energies, the above questions are not merely academic but are extremely relevant to understanding the initial conditions for these collisions. The primary objective of heavy ion collisions at these energies is to investigate the possible formation of a soup of quark gluon matter–often simply called the quark gluon plasma–and a phase transition of the plasma to hadronic matter [2]. The formation of the plasma and indeed the dynamics of any subsequent phase transition to hadronic matter, will depend sensitively on these initial conditions.

In this comment, I will discuss recent work [3–10] which seeks to answer the above questions quantitatively by addressing the problem of initial conditions for nuclear collisions within the the framework of Quantum Chromodynamics (QCD). The center of mass energies of the colliding nuclei at RHIC and LHC are 100 GeV and 2.7 TeV respectively. Since these energies are far greater than the typical energies of nuclear interactions, the appropriate degrees of freedom in describing these relativistic nuclei must be quarks and gluons, whose interactions are described by QCD.

For ultrarelativistic nuclear collisions at central rapidities, the properties of quarks and gluons at very low values of $x \approx k_t/\sqrt{s}$ are relevant. (Note that x is the

light cone momentum fraction of the nuclear momentum carried by the quark or gluon, k_t is its transverse momentum and $\sqrt{s}$ is the center of mass energy). Recently, there has been renewed interest in QCD at small x because of the results of the deeply inelastic electron proton scattering experiments for $Q^2 >> \Lambda^2_{QCD}$ at HERA and the nuclear shadowing experiments at Fermilab and CERN. For an excellent introduction to the field, see Ref. [11]. The results of the HERA experiments show a very rapid rise in parton distributions for $x << 1$ which is explained both by the conventional (operator product expansion), leading twist Double Leading Log approximation [12,13] and the less conventional BFKL equation [15,43]. However, in the asymptotic limit of $x \to 0$, neither of these approximations are correct because they would both violate the unitarity bound on the growth of cross sections at asymptotic energies [14].

These explanations break down completely at very small x because the parton densities become very large and many body effects become important. Consequences of parton "overcrowding" are that two soft partons may recombine to form a harder parton or a parton may be screened by a cloud of surrounding wee partons [16, 17]. These processes inhibit the growth of parton distributions which saturate at some critical x. Indeed, these processes become important in nuclei at larger values of x than in nucleons. This may explain the strong A dependent shadowing seen in the deeply inelastic scattering (DIS) off nuclei at Fermilab and CERN [18].

It may be argued that practically all one needs in order to determine the dynamics after a nuclear collision are the empirical nuclear structure functions at small x [19]. Using the QCD factorization theorem, products of these probabilities of finding a parton in the nucleus may then be convolved with the elementary parton–parton cross sections to determine parton scattering rates after the nuclear collision. However, factorization breaks down at small x (central rapidities) and coherence effects become important. Partons from one nucleus, at relevant transverse momentum scales, do not resolve individual partons from the other nucleus. As in the

quantum theory of scattering, one needs to take the overlap of the *wavefunctions*– or more specifically, the small x Fock component of the nuclear wavefunction to determine the subsequent nuclear evolution.

This question about the nuclear wavefunction is best formulated on the light cone using the method of light cone quantization [20]. One can write down the light cone QCD Hamiltonian, which is separable into a kinetic term and a potential term. Mueller has shown recently that for heavy quarkonia, where the scale of the coupling constant is set by the mass of the "onium" pair, light cone perturbation theory can be used to construct multi–parton eigenstates at small x [22]. Note however, that despite many attempts, which go under the label "Light Front QCD", thus far the light cone approach to *non–perturbative* QCD has only had limited success [24].

We wish to argue that when the density of partons is extremely large, at very low x in a nucleon or in extremely large nuclei, the density of partons sets the scale for the running of the coupling constant. In other words, if

$$\rho = \frac{1}{\pi R^2}\frac{dN_{part}}{dy} >> \Lambda^2_{QCD}\,, \tag{1}$$

then $\alpha_S(\rho) << 1$. Here we will discuss specifically the application of weak coupling techniques in large nuclei at small values of x, with $x << A^{-1/3}$. An intrinsic scale in the problem is set by the quantity $\mu^2 \sim A^{1/3}$ fm^{-2}, which is the valence quark color charge squared per unit area. Since it is the only scale in the problem, the coupling constant will run as a function of this scale [3].

In Section 2, we write down a partition function for the parton distributions at small x in the presence of the valence quarks which play the role of external sources in the problem. The background field for this theory is the non–Abelian analogue of the well known Weizsäcker–Williams field in quantum electrodynamics [25]. The parton distribution functions are formally expressed as correlation functions of a 2-dimensional Euclidean field theory with the effective coupling $\alpha_s\mu$. The correlation functions to each order in α_s, involve an infinite resummation to all orders in $\alpha_s\mu$ [4].

It was hoped that this classical theory would generate a screening mass ($\propto \alpha_s\mu$) which regulates the infrared behavior of the full theory. Recent lattice results [26] suggest however that the classical problem may not be well defined in the infrared. A possible resolution is that the mass scale for the low momentum modes is generated at the quantum level by the high momentum modes [27]. Higher order corrections to the background field are also discussed briefly in Section 2.

Nuclear collisions are addressed in Section 3. Within the above picture, nuclear collisions can be understood as the collision of two Weizsäcker–Williams fields. Since the fields are non–Abelian, the classical gluon field generated after the collision is obtained by solving the non–linear Yang–Mills equations with boundary conditions specified by the Weizsäcker–Williams field of each nucleus [9, 10]. In the central region of the collision, one therefore sees the highly non–perturbative (in $\alpha_s\mu$) evolution of the Weizsäcker–Williams glue (and sea quarks). The time scale for the dissipation of these non–linearities is on the order of $\sim 1/\alpha_s\mu$. On time scales much larger than this time scale, the evolution of these fields can be described by the hydrodynamic scenario put forward by Bjorken. The quantum picture of nuclear collisions is discussed briefly, with particular reference to the "Onium" picture of Mueller.

In Section 4, we will briefly discuss points of commonality as well as difference between the Weizsäcker– Williams model and the above mentioned models as regards their conceptual foundations as well as their predictions for the experiments which will be performed at RHIC and LHC. These include the parton cascade model of Geiger and Muller [28, 29], the various string fragmentation models [30] and the "color capacitor" models [31] which describe particle production in ultrarelativistic nuclear collisions by the QCD analog of the Schwinger mechanism in quantum electrodynamics.

Section 5 will contain our conclusions.

2 Computing parton distributions for a large nucleus

In this section, the problem of calculating the distributions of partons in the nuclear wavefunction is formulated as a many body problem. We work in the infinite momentum frame using the technique of light cone quantization. Our gauge of choice will be light cone gauge $A^+ = 0$. For an excellent discussion of the advantages of light cone quantization, we refer the reader to Ref. [20]. In light cone quantization and light cone gauge, the electromagnetic form factor of the hadron F_2, measured in deeply inelastic scattering experiments, is simply related to parton distributions by the relation [21]

$$F_2(x, Q^2) = \langle \int^{Q^2} d^2k_t dk^+ x\delta(x - \frac{k^+}{P^+}) \sum_{\lambda=\pm} a^\dagger_\lambda a_\lambda \rangle \, . \tag{2}$$

We use light cone co–ordinates ($x^\pm = (t \pm x)/\sqrt{2}$). In the above, P^+ is the momentum of the nucleus, k^+ and k_t are the parton longitudinal and transverse momenta respectively, x is the light cone momentum fraction, Q^2 is the momentum transfer squared from the projectile and $a^\dagger a$ is the number density of partons in momentum space. One only need integrate the calculated distributions up to the scale Q^2 to make comparison with experiment.

2.1 A partition function for wee partons in a large nucleus

In QED, the infinite momentum frame wavefunction of the system with the external source in Eq. (5) is a coherent state [3]. Failing to do the same in QCD, we compute ground state expectation values instead. The partition function for the ground state of the low x partons in the presence of the valence quark external source is

$$Z = < 0|e^{iTP^-}|0 > = \lim_{T\rightarrow i\infty} \sum_N < N|e^{iTP^-}|N >_Q \, . \tag{3}$$

The sum above also includes a sum over the color labels of the sources of color charge (denoted by Q) generated by the valence quarks.

The light cone Hamiltonian P^- (generator of translations in light cone time x^+), in the presence of an external source, can be written as

$$\begin{aligned} P^- &= \frac{1}{4}F_t^2 + \frac{1}{2}\left(\rho_F + D_t \cdot E_t\right)\frac{1}{P^{+2}}\left(\rho_F + D_t \cdot E_t\right) \\ &+ \frac{1}{2}\psi^\dagger (M - \not{P}_t)\frac{1}{P^+}(M + \not{P}_t)\psi\,, \end{aligned} \tag{4}$$

where P^- can be split into kinetic and potential pieces. Here ρ_F is the charge density due to the external source plus the dynamical quarks, A_t and ψ are dynamical components of the vector and spinor fields respectively, $E_t = \partial_- A_t$ is the transverse electric field and F_t^2 is the transverse field strength tensor. See Ref. [3] for more details regarding the above expression and the conventions used.

Valence quarks are predominantly found at large values of x. It is therefore reasonable to assume that they constitute the sources of the external charge seen by the wee partons. This current takes the form

$$J_a^\mu = \delta^{\mu +}\rho_a(x^+, \vec{x}_\perp)\delta(x^-)\,. \tag{5}$$

In the gauge $A^+ = 0$, the static component J^+ is the only large component of the valence quark current. The transverse and minus components are proportional to $1/P^+$ and are therefore small. The current seen by the wee partons is proportional to $\delta(x^-)$ if the valence quarks are Lorentz contracted to a size which is much smaller than a co–moving wee parton's wavelength. This is satisfied if $2Rm/P^+ << 1/xP^+ \Longrightarrow x << 1/Rm \sim A^{-1/3}$ where R is the nuclear radius and m the nucleon mass.

Evaluating the trace in the partition function for quantized sources of color charge is difficult. We simplify the problem by resolving the transverse space as a grid of boxes of size $d^2x_t >> 1/\rho_{val}$ (or equivalently, parton transverse momenta $q_t^2 << \rho_{val}$) which contains a large number of valence quarks and hence a large number of color charges. This allows us to treat the sum over color configurations classically [3]. We average over the color charges by introducing in the path integral

representation of the partition function the Gaussian weight

$$\exp\left\{-\frac{1}{2\mu^2}\int d^2x_t\ \rho^2(x)\right\}\ , \tag{6}$$

where ρ is the color charge density (per unit area) and the parameter μ^2 is the average color charge density squared (per unit area) in units of the coupling constant g. It can be written as

$$\mu^2 = \rho_{val} < Q^2 >\equiv \frac{3A}{\pi R^2}\frac{4}{3}g^2 \sim\ 1.1\ A^{1/3}\ fm^{-2}\,, \tag{7}$$

where $< Q^2 >= 4g^2/3$ is the average color charge squared of a quark.

We can now write the partition function Z in the Light Cone gauge $A_- = 0$ as

$$\begin{aligned} Z &= \int\ [dA_t dA_+][d\psi^\dagger d\psi][d\rho] \\ &\quad \exp\left(iS + ig\int d^4x A_+(x)\delta(x^-)\rho(x) - \frac{1}{2\mu^2}\int d^2x_t\rho^2(0,x_t)\right)\ . \end{aligned} \tag{8}$$

Hence, the result of our manipulations is to introduce a dimensionful parameter $\mu^2 \approx 1.1\ A^{1/3}\ \mathrm{fm}^{-2}$ in the theory. If we impose current conservation, $D_\mu J^\mu = 0$ and integrate over the external sources ρ, we obtain a non–local theory [3] containing modified propagators and vertices.

2.2 The classical background field of a nucleus

Equivalently, one can find the classical background field in the presence of the external sources, compute correlation functions in this background field and finally integrate over the Gaussian random sources.

The equations of motion are

$$D_\mu F^{\mu\nu} = gJ^\nu\ ;\ J^\mu_a = \delta^{+\mu}\rho_a(x^+,x_t)\delta(x^-)\ . \tag{9}$$

The background field which satisfies the above equation of motion is $A^\pm = 0$, $A_i(x) = \theta(x^-)\,\alpha_i(x_t)$. Also, $F_{12} = 0$ and $\nabla\cdot\alpha = g\rho(x_t)$. Because the field strength

$F_{12} = 0$, α_i is a pure gauge: $\tau \cdot \alpha_i = -\frac{1}{ig} U \nabla_i U^\dagger$. Combining the two equations results in the highly non–linear stochastic differential equation

$$\vec{\nabla} \cdot U \vec{\nabla} U^\dagger = -ig^2 \rho(x_t) \,. \tag{10}$$

Here ρ is the surface charge density associated with the current J. There is no dependence on x^- because we have factored out the delta function. The dependence on x^+ goes away because of the extended current conservation law $D_\mu J^\mu = 0$ in the background field.

To compute correlation functions associated with our classical solutions, we must solve the above equation and integrate the rho–dependent gauge fields over all color orientations of the external sheet of charge. In the matrix notation,

$$\begin{aligned} \langle \alpha_i^{\alpha\beta}(x_t) \alpha_j^{\alpha'\beta'}(0) \rangle &= \frac{-1}{g^2} \int [d\rho] \left(U(x_t) \nabla U^\dagger(x_t) \right)_\rho^{\alpha\beta} \left(U(0) \nabla U^\dagger(0) \right)_\rho^{\alpha'\beta'} \\ &\times \exp\left(-\frac{1}{2\mu^2} \int d^2x_t \rho^a(x_\perp) \rho^a(x_\perp) \right) . \end{aligned} \tag{11}$$

The relation between distribution functions and propagators is straightforward and is discussed explicitly in Ref. [5].

$$\frac{1}{\pi R^2} \frac{dN}{dx d^2k_t} = \frac{1}{(2\pi)^3} \frac{1}{x} \int d^2x_t \; e^{ik_t x_t} \; \mathrm{Tr} \; [\langle \alpha_i^{\alpha\beta}(x_t) \alpha_j^{\alpha'\beta'}(0) \rangle] \,, \tag{12}$$

where the trace is over both Lorentz and color indices.

It was believed previously that the distribution function has the general form

$$\frac{1}{\pi R^2} \frac{dN}{dx d^2k_t} = \frac{(N_c^2 - 1)}{\pi^2} \frac{1}{x} \; \frac{1}{\alpha_S} H(k_t^2/\alpha_S^2 \mu^2) \,, \tag{13}$$

where $H(k_t^2/\alpha_S^2\mu^2)$ is a non–trivial function obtained by explicitly solving Eq. (11). It was further believed that in the "weak coupling" limit of $k_t >> g^2\mu$, $H(y) \to 1/y$ and one obtains the Weizsäcker–Williams result

$$\frac{1}{\pi R^2} \frac{dN}{dx d^2q_t} = \frac{\alpha_S \mu^2 (N_c^2 - 1)}{\pi^2} \frac{1}{x q_t^2} \,, \tag{14}$$

scaled by μ^2. It was expected that the function $H(k_t^2/\alpha_s^2\mu^2)$ would have in the strong coupling region the behavior $\alpha_s^2\mu^2/(k_t^2+M_s^2)$ where $M_s \sim \alpha_s\mu$ is a screening mass which would regulate the divergence of the distribution function at small k_t. If such a screening mass did exist, it would provide a simple understanding of saturation already at the classical level.

Unfortunately, this turns out *not* to be the case. Unable to find an analytic solution to Eq. (10) we recently solved the correlation functions numerically on the lattice using the conjugate gradient method [26]. Our preliminary results suggest the following. Weak (strong) coupling on the lattice holds when $0.2g^2\mu L << (>>)1$, where L is the lattice size. In the weak coupling limit, our results indicate a discrete transverse momentum dependence which is of the $1/k_t^2$ Weizsäcker–Williams form. As one increases $g^2\mu L$, there is an additional "transverse" contribution to the correlation function which displays an exponential fall off determined by a "screening mass". However, the amplitude of this term appears to diverge as $\sim (g^2\mu L)^2$. At large values of $g^2\mu L$, it appears that no solutions of the stochastic equations exist and the classical theory is ill defined in the infrared.

Albeit it appears that no infrared stable screening mass is generated at the classical level, such a screening mass may still be generated by quantum fluctuations around the high transverse momentum modes [27]. Another possibility is that the Gaussian weight has a momentum dependence which modifies the infrared behavior of the correlation functions. Finally, the assumption that the source is a delta function on the light cone may be too severe. A way to deal with this problem is suggested in very interesting recent work by Balitsky [33]. These ideas are outside the scope of the present work. We shall assume in the following that the computation of small fluctuations in the background field is not modified seriously by our lattice result (i.e., the theory can, for instance, be regularized by integrating over the random sources with an appropriate weight).

2.3 Quantum corrections to background field

We now outline a procedure (the familiar Dyson–Schwinger expansion [35]) to systematically compute quantum corrections to our background field to all orders. The fully connected two point function is given by the relation

$$\langle\langle AA\rangle\rangle_\rho = \langle\langle A_{cl}\rangle\langle A_{cl}\rangle + \langle A_q A_q\rangle\rangle_\rho \, . \tag{15}$$

In the above, $\langle A_{cl}\rangle$ is the expectation value of the classical field to all orders in $\hbar$. It can be expanded as $\langle A_{cl}\rangle = A_{cl}^{(0)} + A_{cl}^{(1)} + ...$ where $A_{cl}^{(0)}$ is the solution discussed in Section 2.2. The term $\langle A_q A_q\rangle$ is the small fluctuation Green's function computed to each order in the classical field. The symbol $< \cdots >_\rho$ indicates that we have to average over the external sources of color charge with the Gaussian weight described previously. We will briefly discuss the computation of the small fluctuation Green's function and how it may be used to compute the one loop correction to a) the gluon distribution function and b) the classical field $A_{cl}^{(1)}$.

We begin by considering small fluctuations around our classical background, $A_{cl} = A_{cl}^0 + \delta A$. Substituting this in the partition function in Eq. (8), we only keep terms $\mathrm{O}(\delta A^2)$ in the action. The small fluctuations propagator may be computed directly from the action or directly from the small fluctuation Yang–Mills equations, keeping terms linear in δA, and solving the resulting eigenvalue equation. The final expression which involves several subtle features of light cone quantization is quite lengthy and the reader is referred to Ref. [6] for the details.

In Ref. [7], we have used the small fluctuation Green's function in light cone gauge to compute the one loop correction to the background field as well as the one loop contribution to the classical field. The perturbative expression for the gluon distribution function to second order in α_s is

$$\frac{1}{\pi R^2}\frac{dN}{dx d^2k_t} = \frac{\alpha_s \mu^2 (N_c^2-1)}{\pi^2}\frac{1}{x k_t^2}\left\{1 + \frac{2\alpha_s N_c}{\pi}\ln\left(\frac{k_t}{\alpha_s \mu}\right)\ \ln\left(\frac{1}{x}\right)\right\}. \tag{16}$$

Equation (16) contains both $\ln(1/x)$ and $\ln(k_t)$ corrections to the $1/(xk_t^2)$ distribution and they represent the first order contributions to the perturbative expansion for the distribution function. In the kinematical region of validity, these corrections are large. This signals that in order to properly account for the perturbative corrections one has to devise a mechanism to isolate and sum up these leading contributions. This work is in progress [27].

The one loop corrections to the classical background field are computed as follows. We start with the classical equations of motion $D_\mu F_a^{\mu\nu} = gJ_a^\nu$ and expand the full gluon field as $A^\mu = B^\mu + b^\mu$ where B^μ is the background (classical) field, that is $< A^\mu >= B^\mu$, while b^μ is the fluctuation (quantum) field with $< b^\mu >= 0$. Keeping up to quadratic terms in b^μ, one can write equations for the $+,-$ and transverse components of the equations of motion. All the terms involving bilinear products of b^μ in the minus and transverse components of the equations can be shown to vanish and these equations are identical to their classical counterparts. The equation for the $+$ component is

$$-\partial_- \partial^+ B_a^- - (D_i \partial^+ B^i)_a = g j_a^+ + g < J_a^+ > , \tag{17}$$

where the induced current in the above equation is related simply to the small fluctuations propagator $G_{bc}^{ij}(x,y)$ by the equation

$$j_a^+(x) = f_{abc} < b_b^i(x) \partial^+ b_c^i(x) >= i f_{abc} \lim_{y\to x} \frac{\partial}{\partial y^-} G_{bc}^{ii}(x,y) \, . \tag{18}$$

We find that the Fourier transform of $j_a^+(x)$ is $g\tilde{j}_a^+(p) = \Pi_{ab}^{+i}(p) A_b^{i(0)}(p)$, where $\Pi_{ab}^{+i}(p)$ is given by

$$\Pi_{ab}^{+i}(p) = g^2 p^+ p^i \left(\frac{5\Gamma(-\omega)}{16\pi^2} \right) \delta_{ab} \, , \tag{19}$$

which is the standard expression for the $+i$ components of the polarization operator in light cone gauge [42]. We conclude from the above that the modifications to the background field introduced by the quantum fluctuations do not induce extra

terms in the expression for the distribution function [7]. This is consistent with the theorem of Dokshitzer, Diakonov and Troyan [38]. The effect of quantum corrections to the background field can be included by replacing the coupling constant g by the renormalized coupling constant g_R which runs as a function of μ^2. The structure of the background field at one loop remains unchanged.

3 Nuclear collisions of Weizsäcker–Williams fields

In the previous section we discussed the properties of the Weizsäcker–Williams field of a single nucleus. Recently, A. Kovner, L. McLerran and H. Weigert [9, 10] have made significant progress in solving the *classical* problem of the evolution of these fields after the nuclear collision.

Below, we outline very briefly their key results and refer the interested reader to their papers for further details. Before the two nuclei collide (for times $t < 0$), the Yang–Mills equations for the background field of two nuclei on the light cone is simply $A^\pm = 0$ and

$$A^i = \theta(x^-)\theta(-x^+)\alpha_1^i(x_\perp) + \theta(x^+)\theta(-x^-)\alpha_2^i(x_\perp) \tag{20}$$

The two dimensional vector potentials are pure gauges (as in the single nucleus problem!) and for $t < 0$ solve $\nabla \cdot \alpha_{1,2} = g\rho_{1,2}(x_\perp)$. The interesting aspect of this solution is that the classical field configuration does not evolve in time for $t < 0$! This is a consequence of the highly coherent character of the wee parton clouds in the nuclei.

The above solution for $t < 0$ is a fairly straightforward deduction from the single nucleus case. What is very interesting is that the above mentioned authors find a solution to the field equations after the nuclear collision (for $t > 0$). It is given by

$$A^\pm = \pm x^\pm \alpha(\tau, x_\perp)\ ;\ A^i = \alpha_\perp^i(\tau, x_\perp)\,, \tag{21}$$

where $\tau = \sqrt{t^2 - z^2} = \sqrt{2x^+x^-}$. The relation between $A^\pm$ follows from the gauge condition $x^+A^- + x^-A^+ = 0$. This solution only depends on longitudinal boost invariant variable τ and has no dependence on the space-time rapidity variable $y = \frac{1}{2}\ln\frac{x^+}{x^-}$. This suggests that the parton distributions will be boost invariant for all later times. This result therefore justifies Bjorken's ansatz [34] for the subsequent hydrodynamic evolution of the system.

The above ansatz for the background field can be substituted in the Yang–Mills equations to obtain highly non–linear equations for $\alpha(\tau, x_\perp)$ and $\alpha_\perp^i(\tau, x_\perp)$. The detailed expressions are given in Ref. [10]. The initial conditions for the evolution of these equations depend on the single nucleus solutions:

$$\alpha_\perp^i|_{\tau=0} = \alpha_1^i + \alpha_2^i \;\; ; \;\; \alpha|_{\tau=0} = \frac{ig}{2}\,[\alpha_1^i, \alpha_2^i]\,, \tag{22}$$

where $\alpha_{1,2}^i$ are the background fields for the two nuclei.

The Yang–Mills equations with the above boundary conditions are solved perturbatively, order by order, by expanding the fields in powers of the valence quark charge density ρ. For asymptotically large τ, Kovner et al. find that a gauge transform of the fields α and $\alpha_\perp^i$ (denoted here by ϵ and $\epsilon_\perp^i$ respectively) have the form

$$\begin{aligned} \epsilon^a(\tau, x_\perp) &= \int \frac{d^2k_\perp}{(2\pi)^2}\frac{1}{\sqrt{2\omega}}\left\{a_1^a(\vec{k}_\perp)\frac{1}{\tau^{3/2}}e^{ik_\perp\cdot x_\perp - i\omega\tau} + C.C.\right\} \\ \epsilon^{\vec{a},i}(\tau, x_\perp) &= \int \frac{d^2k_\perp}{(2\pi)^2}\kappa^i\frac{1}{\sqrt{2\omega}}\left\{a_2^a(k_\perp)\frac{1}{\tau^{1/2}}e^{ik_\perp x_\perp - i\omega\tau} + C.C.\right\}\,, \end{aligned} \tag{23}$$

where a_1 and a_2 can be expressed in terms of the ρ fields. In this equation, the frequency is $\omega = \mid k_\perp \mid$ and the vector $\kappa^i = \epsilon^{ij}k^j/\omega$. The notation C. C. denotes complex conjugate.

With the above form for the fields, the expressions for the parton number densities is straightforward. For late times, near $z = 0$, one obtains [41]

$$\frac{dN}{dyd^2k_\perp} = \frac{1}{(2\pi)^3}\sum_{i,a} \mid a_i^a(k_\perp)\mid^2 \tag{24}$$

Averaging over the ρ fields with the Gaussian weight in Eq. (6), one obtains the following result for the gluon distribution at late times after the nuclear collision:

$$\frac{1}{\pi R^2}\frac{dN}{dyd^2k_\perp} = \frac{16\alpha_s^3}{\pi^2}N_c\,(N_c^2-1)\,\frac{\mu^4}{k_t^4}\,\ln(\frac{k_t}{\alpha_s\mu})\,. \tag{25}$$

As suggested by the logarithm in the above equation, the transverse momentum integrals are infrared divergent. They are cut off by a mass scale $\alpha_s\mu$ which was believed to result from the non–perturbative behavior of the classical fields. However, as we have discussed earlier, our recent lattice calculations suggest instead that the theory is not well defined in the infrared. Any such dynamically generated mass will therefore arise only at the quantum level.

This brings up a related issue. Thus far we have only discussed the dynamical evolution of the classical fields. What about quantum effects? One way to include these is to do what we did for a single nucleus–look at small fluctuations around the background field of two nuclei [37]. The background field in this case is much more complicated than in the single nucleus case and the quantum problem is significantly more difficult. Another approach is to consider what quantum effects do to the coherence of the initial wavepacket.

In this regard, A. H. Mueller's [22] formulation of the low x problem is relevant. He considers an "Onium" (heavy quark–anti-quark) state of mass M for which $\alpha_S(M) \ll 1$. In weak coupling, the $n-$ gluon component of the onium wavefunction obeys an integral equation whose kernel in the leading logarithmic and large N_c limit is precisely the BFKL kernel [15]. The derivation relies on a picture in which the onium state produces a cascade of soft gluons strongly ordered in their longitudinal momentum; the i–th emitted gluon has a longitudinal momentum much smaller than the $i-1$–th.

In the large N_c limit, the n gluons can be represented as a collection of n–dipoles. Hence, in high energy onium–onium scattering, the cross section is proportional to the product of the number of dipoles in each onium state times the dipole–dipole

scattering cross section [23]. This cross section is given by two gluon exchange (the pomeron). More complicated exchanges involving multi–pomeron exchange have been studied recently by Salam [36]. However, despite the mathematical elegance and simple interpretation of the onium approach, it is unclear whether it can be extended to nuclei.

4 Parton cascades and color capacitors

In this section we will briefly discuss, in relation to the model discussed in earlier sections, some other attempts to model the initial conditions for ultrarelativistic heavy ion collisions. They may be broadly (and somewhat imprecisely) classified as follows: a) perturbative QCD based models which assume the factorization theorem and incoherent multiple scattering to construct a spacetime picture of the nuclear collision, and b) non–perturbative models where particle production is based on string fragmentation or pair creation in strong color fields.

Among perturbative QCD based models, the parton cascade model of Geiger and Müller [28, 29] has been applied extensively to study various features of heavy ion collisions. The evolution of *classical* phase space distributions of the partons is specified by a transport equation of the form

$$\left[\frac{\partial}{\partial t} - \vec{v}\cdot\frac{\partial}{\partial \vec{r}}\right] F_a(\vec{p},\vec{r},t) = C_a(\vec{p},\vec{r},t)\,, \tag{26}$$

where F_a are the *classical* phase space distributions for particle type a and C_a is the corresponding collision integral. The matrix elements in the collision integral are computed from the relevant tree level diagrams in perturbative QCD.

The initial conditions in the parton cascade model are specified at some initial time $t = t_0$ by the distribution $F_a(\vec{p},\vec{r},t=t_0) = P_a(\vec{p},\vec{P})R_a(\vec{r},\vec{R})$. The momentum distribution $P_a(\vec{p},\vec{P}) = f_a(x,Q_0^2)g(p_t)$ is decomposed into an uncorrelated product of longitudinal and transverse momentum distributions respectively, where $f_a(x,Q_0^2)$

is the nuclear parton distribution which is taken from Deep Inelastic Scattering experiments on nuclei at the relevant Q_0^2 and $g(p_t)$ is parametrized by a Gaussian fit to proton–proton scattering data. The spatial distribution of the partons is described by a convolution of a Woods–Saxon distribution of nucleons in the nucleus and an exponential distribution of individual partons within each nucleon. Details regarding both initial conditions may be found in Ref. [29]. Another model which takes as input the perturbative QCD cross sections is the HIJING model [39, 40] which describes nuclear scattering in an eikonal formalism which convolves binary nucleon collisions. In both models, detailed predictions have been made for various observables at RHIC–in particular, for mini–jet production.

As suggested by the above, both models make assumptions which are not necessarily motivated by perturbative QCD. To an extent, this is inevitable because one is forced to model the soft physics. Where these approaches differ significantly from the Weizsäcker–Williams approach is in the factorization assumption, namely, that partons from one nucleus resolve individual partons of the other in each hard scattering. We have argued that the small x partons which dominate the physics of the central region instead have highly coherent "wave– like" interactions. This results in a vastly different space–time picture for the nuclear collision–at least for the very primordial stage of the nuclear collision.

Naturally, the predictions of these models will differ significantly from the Weizsäcker–Williams model. For instance, because of the intrinsic $p_t \sim \mu$ carried by the Weizsäcker–Williams (or "equivalent") gluons, gluon production is enhanced by a factor α_s relative to the lowest order $gg \to gg$ mini–jet process in a cascade. A simple explanation for this enhancement is that because the valence quarks absorb the "recoil", two off–shell equivalent gluons can combine to produce an on–shell gluon. This will impact significantly the many signatures to be studied at RHIC and LHC such as jet production and dilepton and photon production. Further, the intrinsic p_t of the gluons ensures that intrinsic charm and strangeness production is

significantly larger in the Weizsäcker–Williams model [5].

The non–perturbative models [30] primarily attempt to describe the soft physics in ultrarelativistic nuclear collisions so it is not clear that there is much overlap with the Weizsäcker–Williams model. However, the latter does provide some insight into one of these approaches, which we shall dub the "color capacitor" approach. Here it is assumed that the nuclei produce a homogeneous chromo–electric field which produces particles non–perturbatively by a mechanism analogous to the Schwinger mechanism for strong electromagnetic fields. The evolution of these fields (including back-reaction) is determined by a Boltzmann–like equation where the source term now is given by the pair production rate [31, 32].

An important assumption in these color capacitor models is that of homogeneity of the initial field configurations. However, the results discussed in the previous section suggest that the Yang–Mills fields are highly non–linear and inhomogeneous. The time scale $\tau >> 1/\alpha_s\mu$ is the time scale for the dissipation of the non–linearities in the fields. It would be interesting to see how the solutions to the transport equations are modified for initial conditions given by the inhomogeneous Weizsäcker–Williams field configurations.

5 Conclusions

We have described in this Comment a QCD based approach to describe the initial conditions for ultrarelativistic nuclear collisions. The central region of these collisions is dominated by "wee" partons which carry only a small fraction of the nuclear momentum. We have argued that for very large nuclei these partons are only weakly coupled to each other. However, due to their large density, many body effects are extremely important. The classical behavior of these quanta (the QCD analogue of the Weizsäcker–Williams equivalent photons) can be described by an effective two dimensional field theory. Quantum effects are treated by constructing

the small fluctuations propagator in the background field of these quanta.

An important objective of this approach is to understand if there is a "Lipatov region" in nuclei where the parton densities grow rapidly and if the shadowing of parton distributions in nuclei can be understood to result from the precocious onset of parton screening. It is probable that deep inelastic scattering experiments off large nuclei will be performed at HERA in the near future [44]. If so, one may expect unprecedentedly high parton densities and interesting and perhaps unexpected phenomena in these experiments.

These DIS experiments on nuclei at HERA would nicely complement the heavy ion program at RHIC and especially LHC since they probe the same range of Bjorken x. The results of these experiments would therefore place strong bounds on mini–jet multiplicities and other signatures of nuclear collisions. Note that these observables are extremely sensitive to the initial parton distributions (for a discussion, see Ref. [19]). However, to fully understand the dynamics of nuclear collisions at central rapidities, we have to understand the initial conditions ab initio–preferably in a QCD based approach like the one discussed in this paper.

At the moment there are still many open questions which remain unresolved. An empirical question is with regard to the applicability of weak coupling methods to large nuclei. Is the bare parameter $\mu^2 \sim A^{1/3}$ fm^{-2} large enough? One may argue on the basis of Renormalization Group arguments that this parameter should effectively be larger and should grow with the increasing parton density at small x. However, these arguments are not rigorous at this stage.

A more serious problem is suggested by recent lattice simulations of the 2–D effective field theory which show that the classical correlation functions diverge quadratically with the lattice size L. Identifying why this divergence occurs and how the background field may be modified accordingly needs to be resolved satisfactorily. Finally, the problem of Lipatov enhancement and saturation in nuclear parton distributions is not yet settled.

Despite the many technical problems that remain, there is much cause for optimism since it appears now that the problem of initial conditions in ultrarelativistic nuclear collisions can be treated systematically in a QCD based approach. Because the various empirical signatures depend sensitively on the initial conditions, one may hope to identify and interpret the elusive quark gluon plasma in ultrarelativistic nuclear collisions at RHIC and LHC early in the next millenium.

Acknowledgments

Foremost, I would like to acknowledge my debt to my collaborators A. Ayala, R. Gavai, J. Jalilian–Marian, A. Kovner, H. Weigert and especially L. McLerran. I would also like to acknowledge useful discussions with I. Balitsky, B. Müller, R. Narayanan, J. Qiu, T. Schaefer and M. Strikman. Finally, I would like to thank J. Kapusta for his patience and good humor.

Research supported by the U.S. Department of Energy under grants No. No. DOE Nuclear DE–FG06–90ER–40561.

References

[1] H. Cheng and T. T. Wu, *Phys. Rev. Lett.* **24** 759 (1970).

[2] see for instance the proceedings of *Quark Matter 95*, Ed. A. M. Poskanzer, J. W. Harris and L. S. Schroeder, North Holland Publishers.

[3] L. McLerran and R. Venugopalan, *Phys. Rev.* **D49** 2233 (1994).

[4] L. McLerran and R. Venugopalan, *Phys. Rev.* **D49** 3352 (1994).

[5] L. McLerran and R. Venugopalan, *Phys. Rev.* **D50** 2225 (1994).

[6] A. Ayala, J. Jalilian–Marian, L. McLerran and R. Venugopalan, *Phys. Rev.* **D52** 2935 (1995) .

[7] A. Ayala, J. Jalilian–Marian, L. McLerran and R. Venugopalan, *Phys. Rev.* **D53** 458 (1996) .

[8] R. Venugopalan, *Nucl. Phys.* **A590** 147c (1995).

[9] A. Kovner, L McLerran and H. Weigert, *Phys. Rev.* **D52** 6231 (1995).

[10] A. Kovner, L McLerran and H. Weigert, *Phys. Rev.* **D52** 3809 (1995).

[11] B. Badelek, K. Charchula, M. Krawczyk and J. Kwieciński, *Rev. of Mod. Phys.* **64** 927 (1992).

[12] D. J. Gross, *Phys. Rev. Lett.* **32** 1071 (1974).

[13] R. D. Ball and S. Forte, *Phys. Lett.* **B351** 313 (1995).

[14] M. Froissart, *Phys. Rev.* **123** 1053 (1961).

[15] E.A. Kuraev, L.N. Lipatov and Y.S. Fadin, *Zh. Eksp. Teor. Fiz* **72**, 3 (1977) (*Sov. Phys. JETP* **45**, 1 (1977)); I.A. Balitsky and L.N. Lipatov, *Sov. J. Nucl. Phys.* **28** (1978) 822.

[16] L.V. Gribov, E.M. Levin and M.G. Ryskin, *Phys. Rep.* **100** 1 (1983); E. Laenen and E. M. Levin, *Nucl. Phys.* **B451** 207 (1995).

[17] A. H. Mueller and J. Qiu, *Nucl. Phys.* **B268** 427 (1986).

[18] T. Carroll, A. Witzmann and H. Melanson, in *Proc. XXVIII Rencontres de Moriond*, Les Arcs, Savoie, France 1993.

[19] K. Eskola, *Nucl. Phys.* **A590** 383c (1995).

[20] S. J. Brodsky and H. -C. Pauli, *Lectures at 30th Schladming Winter School in Particle Physics*, Schladming, Austria, March 1991.

[21] Yu. L. Dokshitzer, V. A. Khoze, A. H. Mueller and S. I. Troyan, *Basics of Perturbative QCD*, Editions Frontiers, 1991.

[22] A. H. Mueller, *Nucl. Phys.* **B415** 373 (1994); *Nucl. Phys.* **437** 107 (1995).

[23] A. H. Mueller and B. Patel, *Nucl. Phys.* **B425** 471 (1994).

[24] M. Burkardt, submitted to *Adv. Nucl. Phys.*, hep–ph/9505259

[25] J. D. Jackson, *Classical Electrodynamics*, Section 15.4, John Wiley Publishing Co.

[26] R. Gavai and R. Venugopalan, to be published.

[27] A. Kovner, L. McLerran, H. Weigert and J. Jalilian–Marian, private communication.

[28] K. Geiger, B. Müller, *Nucl. Phys.* **B369** 600 (1992).

[29] K. Geiger, *Phys. Rep.* **258** 237 (1995).

[30] A. Capella, U. Sukhatme, C-I. Tan and J. Tran Thanh Van, *Phys. Rep.* **236** (1994); B. Andersson, G. Gustafson, G. Ingelman and T. Sjöstrand, *Phys. Rep.* **97** 31 (1983).

[31] J. M. Eisenberg, LBL–34831, *Proceedings of Workshop on Pre–Equilibrium Parton Dynamics*, Berkeley, August 23–September 3, 1993.

[32] F. Cooper, J. M. Eisenberg, Y. Kluger, E. Mottola, B. Svetitsky, *Phys. Rev.* **D 48** 190 (1993).

[33] I. Balitsky, preprint MIT–CTP–2470, september 1995.

[34] J. D. Bjorken, *Phys. Rev.* **D27** 140 (1983).

[35] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics*, Addison–Wesley Publishing Co., Inc. 1989.

[36] G. P. Salam, preprint Cavendish–HEP–95/05, hep–ph/9509353

[37] A. Makhlin, *Phys. Rev.* **C52** 995 (1995).

[38] Yu. Dokshitzer, D. Diakonov and S. Troyan, *Phys. Rep.* **58** 270 (1980).

[39] K. Eskola and Xin-nian Wang, *Phys. Rev.* **D49** 1284 (1994).

[40] M. Gyulassy and Xin-nian Wang, *Comput. Phys. Commun.* **83** 307 (1994).

[41] J.-P. Blaizot and A. H. Mueller, *Nucl. Phys.* **B289** 847 (1987).

[42] G. Leibbrandt, *Rev. of Mod. Phys.* **59** 1067 (1987).

[43] V. Del Duca, DESY preprint, hep-ph/9503226.

[44] For a discussion, see proceedings of the workshop on *Nuclei at HERA and Heavy Ion physics*, Ed. S. Gavin and M. Strikman, Brookhaven, november 1995.